

DETERMINATION OF OPTIMUM PARAMETERS IN SOLVING NEUTRON DIFFUSION EQUATION USING FINITE DIFFERENCE METHOD

Imelda Ariani* and Doddy Kastanya**

ABSTRACT

DETERMINATION OF OPTIMUM PARAMETERS IN SOLVING NEUTRON DIFFUSION EQUATION USING FINITE DIFFERENCE METHOD. A three-dimensional, multi-group neutron diffusion equation solver based on the finite difference method (FDM) has been developed in FORTRAN. The development of this code is a part of preliminary efforts to develop a full scale pressurized water reactor core simulator which will also has the capability of performing fuel loading pattern optimizations. The solver has been verified against the IAEA 3-D PWR benchmark problem. To solve the FDM representation of the diffusion equation, a nested (outer-inner) iteration scheme is employed with a Red-Black Line SOR as the inner iteration solver. Algorithms which can automatically determine optimum parameters needed in the outer-inner iteration calculations have also been implemented such that the code can solve the diffusion equation optimally.

Keywords: pressurized water reactor simulator, neutron diffusion equation, finite difference method.

ABSTRAK

PENENTUAN PARAMETER-PARAMETER OPTIMAL DALAM MENYELESAIKAN PERSAMAAN DIFUSI NEUTRON MENGGUNAKAN METODE BEDA HINGGGA. Sebuah program komputer untuk menyelesaikan persamaan difusi neutron dalam ruang tiga dimensi, energi banyak kelompok dan berbasis metoda beda hingga telah dikembangkan dengan menggunakan bahasa pemrograman FORTRAN. Upaya ini dilakukan sebagai tahap awal pengembangan suatu program simulator teras reaktor air tekan yang nantinya akan dilengkapi dengan kemampuan untuk melakukan optimasi penyusunan elemen bahan bakar di dalam teras. Program komputer ini telah diuji menggunakan 3-D PWR IAEA *benchmark*. Untuk menyelesaikan persamaan difusi neutron dengan metode beda hingga, diterapkan metode iterasi beranting dengan *Red-Black Line SOR* untuk perhitungan paling dalam. Algoritma yang dapat secara otomatis menghitung parameter-parameter optimal untuk metode iterasi beranting ini telah diprogramkan, sehingga kode komputer ini dapat menyelesaikan persamaan difusi neutron secara optimal.

Kata kunci: reaktor air tekan simulator, persamaan difusi neutron, metode beda hingga.

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INTRODUCTION

An accurate and computationally efficient code to simulate the behavior of a PWR core is an essential tool for a nuclear engineer to perform core design calculations and analysis. The currently available code in P2SRM which is able to perform a PWR core calculation is SRAC'95 [2] which uses the CITATION module for its core-level neutronic calculations. The CITATION module solves the multi-group diffusion equations using a finite difference method. When used with a coarse mesh size, the FDM method provides inaccurate solution to the diffusion equation. In addition to the lack of fidelity of the solver, no thermal hydraulic feedbacks essential in analyzing a real/representative PWR is available from the CITATION module. Therefore, there is a plan to develop a PWR core simulator which is more accurate and computationally efficient than SRAC'95 utilizing nodal based solver. The methodology will be similar to what is currently used in modern neutronic codes. As a preliminary step, a 3D (XYZ) diffusion equation solver based on the finite difference method has been developed in FORTRAN language.

FDM ITERATION STRATEGY

A box scheme (material centered) FDM is utilized. The FDM requires solution of a large matrix corresponding to the FDM representation of the multi-group diffusion equation. Much work has been done on formulating, understanding, and implementing the iterative solution of the large, sparse matrix system. An outer-inner iterative strategy is utilized in this code. The outer iteration refers to the iteration needed to update the fission source term while the inner iteration approximately solves the resulting fixed source problem.

The multi-group neutron diffusion equation for node l after applying Fick's law approximation can be written as (for simplicity, only the x-direction is shown here):

$$\frac{d}{dx} j_{gx} + A_g \bar{\mathbf{F}}_{gx}(x) = \bar{Q}_{gx}(x) \quad (1)$$

The FDM representation of this equation in 3-D Cartesian geometry with homogeneous node l can be expressed as:

$$\sum_{l'=1}^L C_g^{l,l'} \bar{\mathbf{F}}_g^{l'} + A_g^l \bar{\mathbf{F}}_g^l = \bar{Q}_g^l \quad (2)$$

where the non-zero values of the coupling coefficients are obtained via:

$$\bar{J}_{gx^+}^{l,FDM} = -\frac{D_{gx^+}^{l,FDM}}{0.5(\Delta x^l + \Delta x^{l+1})} [\bar{\mathbf{f}}_g^{l+1} - \bar{\mathbf{f}}_g^l] \text{ and } D_{gx^+}^{l,FDM} = \frac{D_g^l D_g^{l+1} (\Delta x^l + \Delta x^{l+1})}{D_g^l \Delta x^l + D_g^{l+1} \Delta x^{l+1}} \quad (3)$$

and L denotes the total number of nodes. Substituting definitions for A_g^l and \bar{Q}_g^l into Eq.(2) and rearranging terms, we obtain:

$$\sum_{l' \neq l}^L C_g^{l,l'} \bar{\mathbf{f}}_g^{l'} + (\Sigma_{lg}^l - \Sigma_{sgg}^l + C_g^{l,l}) \bar{\mathbf{f}}_g^l - \sum_{g' \neq g}^G \Sigma_{sgg'}^l \bar{\mathbf{f}}_{g'}^l = \frac{C_g^l}{k} \sum_{g'=1}^G \mathbf{n} \bar{\Sigma}_{fg'}^l \bar{\mathbf{f}}_{g'}^l \quad (4)$$

which can be written in terms of matrix notation spanning the spatial domain as:

$$\bar{\bar{A}}_g \bar{\mathbf{f}}_g - \sum_{g' \neq g}^G \bar{\bar{\Sigma}}_{sgg'} \bar{\mathbf{f}}_{g'} = \frac{1}{k} \bar{\bar{C}}_g \sum_{g'=1}^G \mathbf{n} \bar{\bar{\Sigma}}_{fg'} \bar{\mathbf{f}}_{g'} \quad (5)$$

The matrix $\bar{\bar{A}}_g$ has a seven-banded structure for 3-D Cartesian geometry. The G(LxL) matrix systems expressed in Eq.(5) can be collected to write the single GLxGL matrix system:

$$\bar{\bar{A}} \bar{\mathbf{f}} = \frac{1}{k} \bar{\bar{F}} \bar{\mathbf{f}}, \quad (6)$$

where $\bar{\bar{A}}$ is a block lower triangular matrix assuming there is no upscattering term.

The outer-inner iteration process is summarized as follows. Given an arbitrary initial vector $\bar{\mathbf{f}}^{(0)}$, the outer iterations generate successive estimates for the flux vector $\bar{\mathbf{f}}$ from

$$\bar{\mathbf{f}}^{(q)} = \frac{1}{k^{(q-1)}} \bar{\bar{A}}^{-1} (\bar{\bar{F}} \bar{\mathbf{f}}^{(q-1)}). \quad (7)$$

The iterative matrix associated with the outer iteration is

$$\bar{\bar{Q}} = \bar{\bar{A}}^{-1} \bar{\bar{F}}. \quad (8)$$

The property of the iterative matrix $\bar{\bar{Q}}$ has a significant role in determining the convergence rate of the power iterations [3], *i.e.* the outer-inner iterations.

To solve Eq.(7), we need to take advantage of the structure of the $\overline{\overline{A}}$ matrix. For two energy groups (G=2), energy group decoupling is accomplished by solving for the fast and thermal groups in successive manner. This implies that the following system of linear equation for each energy group:

$$\overline{\overline{A}}_g \overline{\mathbf{f}}_g^{(q)} = \overline{\mathbf{S}}_g^{(q)} \quad (9)$$

where

$$\overline{\mathbf{S}}_g^{(q)} = \sum_{g' \neq g}^G \overline{\overline{\Sigma}}_{sgg'} \overline{\mathbf{f}}_{g'}^{(q)} + \frac{1}{k^{(q-1)}} \overline{\overline{C}}_g \sum_{g'=1}^G \mathbf{n} \overline{\overline{\Sigma}}_{fg'} \overline{\mathbf{f}}_{g'}^{(q-1)} \quad (10)$$

needs to be solved.

Inner Iteration Acceleration

To solve Eq.(9), we introduced inner iterations which employ a Red-Black Line SOR. Mathematically, this multi-splitting method can be expressed as follows. Note that the energy group g and outer iteration count q dependence have been suppressed for clarity.

$$\overline{\mathbf{F}} = \oplus \overline{\mathbf{F}}_p \text{ where } \overline{\mathbf{F}}_p \text{ spans nodes of color } p \quad (11)$$

$$\overline{\mathbf{f}}_p^{(m+1)} = \overline{\overline{B}}_p^{-1} \left[\overline{\mathbf{S}} + \sum_{p'=1}^{p-1} \overline{\overline{C}}_{pp'} \overline{\tilde{\mathbf{f}}}_{p'}^{(m+1)} + \sum_{p'=p+1}^P \overline{\overline{C}}_{pp'} \overline{\tilde{\mathbf{f}}}_{p'}^{(m)} \right] \text{ for } p=1,2,\dots,P \quad (12)$$

$$\overline{\overline{A}} = \oplus \overline{\overline{A}}_p \quad (13)$$

$$\overline{\overline{A}}_p = \overline{\overline{B}}_p - \sum_{p' \neq p}^P \overline{\overline{C}}_{pp'} \text{ for } p=1,2,\dots,P \quad (14)$$

$$\overline{\tilde{\mathbf{f}}}_p^{(m+1)} = \overline{\tilde{\mathbf{f}}}_p^{(m)} + \mathbf{w} \left(\overline{\tilde{\mathbf{f}}}_p^{(m+1)} - \overline{\tilde{\mathbf{f}}}_p^{(m)} \right) \quad (15)$$

$\overline{\overline{B}}_p$ is a square matrix and has a tridiagonal structure which implies that $\overline{\overline{B}}_p^{-1}$ evaluation in Eq.(12) is simple. A total of ΔN_I inner iterations per outer iteration is completed. The ΔN_I value is determined such that the specified relative error reduction from the 0th iterative error for the inner iterations is achieved.

The ω and ΔN_I values are energy group dependent and need to be determined beforehand. It is assumed that the iterative matrix associated with the inner iterative

method is symmetrizable. The ω value can then be expressed in terms of the spectral radius of the associated Gauss Seidel iteration matrix $\mathbf{r}(\overline{\overline{\mathbf{L}}^{GS}})$:

$$\mathbf{w} = \frac{2}{1 + \left(1 - \mathbf{r}(\overline{\overline{\mathbf{L}}^{GS}})\right)^{1/2}} \quad (16)$$

$\overline{\overline{\mathbf{L}}^{GS}} = \overline{\overline{\mathbf{L}}^{SOR}}(\mathbf{w})$ with $\mathbf{w}=1$. Therefore, the calculation of the spectral radius of the associated Gauss-Seidel iterative matrix is the basis of this procedure. The following steps are completed to determine ω value based upon the DIF3D methodology [1]. The steps are done for each energy group.

Step 1: Starting with an arbitrary non-negative initial guess vector $\overline{\overline{\mathbf{x}}^{(0)}}$, complete at least ten Gauss-Seidel iterations to solve $\overline{\overline{\mathbf{A}}}\overline{\overline{\mathbf{x}}} = \overline{\overline{\mathbf{0}}}$

Step 2: Following each iteration with $m > 10$, estimate the upper and lower bounds of the spectral radii:

$$\mathbf{I}^{(m)} = \frac{\langle \overline{\overline{\mathbf{x}}^{(m)}}, \overline{\overline{\mathbf{x}}^{(m)}} \rangle}{\langle \overline{\overline{\mathbf{x}}^{(m)}}, \overline{\overline{\mathbf{x}}^{(m-1)}} \rangle}, \quad \overline{\overline{\mathbf{I}}}^{(m)} = \text{MAX}_i \left| \frac{\mathbf{x}_i^{(m)}}{\mathbf{x}_i^{(m-1)}} \right|, \quad \underline{\mathbf{I}}^{(m)} = \text{MIN}_i \left| \frac{\mathbf{x}_i^{(m)}}{\mathbf{x}_i^{(m-1)}} \right|$$

and compute the corresponding relaxation factors:

$$\mathbf{w}^{(m)} = \frac{2}{1 + \left(1 - \mathbf{I}^{(m)}\right)^{1/2}}, \quad \overline{\overline{\mathbf{w}}}^{(m)} = \frac{2}{1 + \left(1 - \overline{\overline{\mathbf{I}}}^{(m)}\right)^{1/2}}, \quad \underline{\mathbf{w}}^{(m)} = \frac{2}{1 + \left(1 - \underline{\mathbf{I}}^{(m)}\right)^{1/2}}$$

Step 3: Terminate iteration when either $\left| \overline{\overline{\mathbf{w}}}^{(m)} - \underline{\mathbf{w}}^{(m)} \right| < \frac{2 - \mathbf{w}^{(m)}}{5}$ or m equals a

specified upper limit. The optimum ω is set to $\mathbf{w}^{(m)}$. This test forces a tighter convergence of ω when $\mathbf{r}(\overline{\overline{\mathbf{L}}^{GS}})$ is close to unity.

Step 4: Determine the number of inner iterations required for each outer iteration ΔN_I such that the value of ΔN_I satisfies

$$\left\| \left(\overline{\overline{\mathbf{L}}^{SOR}}(\mathbf{w}) \right)^{\Delta N_I} \bullet \overline{\overline{\mathbf{L}}^{GS}} \right\| = \left(t_{2\Delta N_I}^2 + t_{2\Delta N_I}^2 \right)^{1/2} \leq \mathbf{e}_{in}$$

where

$$t_{\Delta N_l} = (\mathbf{w}-1)^{\frac{\Delta N_l-1}{2}} \left(\mathbf{r}(\overline{\overline{L}}^{GS}) \right)^{1/2} \left[1 + (\Delta N_l - 1) \left(1 - \mathbf{r}(\overline{\overline{L}}^{GS}) \right)^{1/2} \right]$$

and \mathbf{e}_{in} denotes the desired relative error reduction from the initial iteration to the end of ΔN_l -th iteration. It is suggested that a very small value of \mathbf{e}_{in} not to be used since it can force excessive inner iterations.

The automated determination of the optimum over-relaxation parameter relieves users from the burden of the trial and error manner of specifying optimum parameters for a large variety of reactor models. In addition, substantial computational time can be saved since the checking of inner iterations' convergence is not needed anymore due to the use of predetermined inner iterations for each energy group.

Outer Iteration Acceleration

The outer iteration defined by Eq.(7) is slow to converge since the dominance ratio (the ratio of the moduli of the two largest eigenvalues) of the iterative matrix in Eq.(8) is close to one. Several methods have been developed to address the issue of convergence acceleration for the fission source iterations. The simplest technique, known as the eigenvalue shift [4] modifies the matrix system of Eq.(6) as follows:

$$\left(\overline{\overline{A}} - \frac{1}{k_s} \overline{\overline{F}} \right) \overline{\overline{\mathbf{f}}} = \left(\frac{1}{k} - \frac{1}{k_s} \right) \overline{\overline{F}} \overline{\overline{\mathbf{f}}} \quad (17)$$

where k_s is a constant selected according to certain restrictions. Eq.(6) is an eigenvalue matrix system where we seek the eigenfunction solution associated with the maximum value of $\left(\frac{1}{k} - \frac{1}{k_s} \right)^{-1}$ which is identical to the flux solution associated with the maximum value of k of Eq.(6). The only restriction is that the modulus of k_s be larger than the modulus of k . The dominance ratio, \mathbf{r}_s , of the modified eigenvalue problem becomes

$$\mathbf{r}_s = \left| \frac{1}{k_1} - \frac{1}{k_s} \right| \left/ \left| \frac{1}{k_2} - \frac{1}{k_s} \right| \right., \quad (18)$$

where k_1 and k_2 are the largest and second largest eigenvalues of the original matrix system. Using the eigenvalue shift, the outer iteration of Eq.(7) can be modified into

$$\bar{\mathbf{f}}^{(q)} = \left(\frac{1}{k^{(q-1)}} - \frac{1}{k_s} \right) \left(\bar{\mathbf{A}} - \frac{1}{k_s} \bar{\mathbf{F}} \right)^{-1} \left(\bar{\mathbf{F}} \bar{\mathbf{f}}^{(q-1)} \right) \quad (19)$$

Similarly, the flux iteration matrix system of Eq.(9) can be written as:

$$\left(\bar{\mathbf{A}}_g - \frac{1}{k_s} \bar{\mathbf{F}}_g \right) \bar{\mathbf{f}}_g^{(q)} = \bar{\mathbf{S}}_g^{(q)} \quad (20)$$

where

$$\bar{\mathbf{S}}_g^{(q)} = \sum_{g' \neq g}^G \bar{\Sigma}_{sgg'} \bar{\mathbf{f}}_{g'}^{(q)} + \left(\frac{1}{k^{(q-1)}} - \frac{1}{k_s} \right) \bar{\mathbf{C}}_g \sum_{g'=1}^G \bar{\mathbf{n}}_{fg'} \bar{\mathbf{f}}_{g'}^{(q-1)} \quad (21)$$

It can be seen from Eq.(18) that the optimum shift for the outer iteration is equal to the true eigenvalue k_1 which results in a dominance ratio of zero. However, the shift also results in the modified flux iteration matrix system of Eq.(9) being singular. Thus, there is a trade-off between savings in the outer iterations versus additional work incurred in the inner iteration solution. Parametric studies are typically performed to determine a range of optimal shift values, k_s , valid over a wide-span of reactor problems. Result from literatures and defining $k_s = k + \Delta k^{\text{shift}}$ suggest the acceptable range of shift of $0.01 < \Delta k^{\text{shift}} < 0.1$.

For our code, Δk^{shift} is fixed throughout the iteration procedure. Additional acceleration of the outer iteration fission source is performed by employing a simple extrapolation based on successive iteration of the fission source:

$$\bar{\mathbf{S}}_g^{(q)} = \bar{\mathbf{S}}_g^{(q-1)} + \mathbf{a} \left(\bar{\mathbf{S}}_g^{(q*)} - \bar{\mathbf{S}}_g^{(q-1)} \right) \quad (22)$$

where $\bar{\mathbf{S}}_g^{(q*)}$ is the unextrapolated fission source determined from Eq.(21) and \mathbf{a} is the extrapolation parameter. The value of \mathbf{a} is determined based on parametric studies with the intent of minimizing the number of outer iterations required to achieve convergence.

RESULTS

The current FDM-based code was benchmarked using the ANL-ID.11-A1 three-dimensional benchmark problem. This problem is also known as the 3D IAEA benchmark problem[†]. The problem is designed to provide a severe test for the capabilities of coarse mesh methods and flux synthesis approximations. The three dimensional configuration of the problem is shown in Figure 1. It is assumed that there is no incoming current at the external boundaries. Reflective boundary conditions at the internal boundaries are applied. The two group constants are shown in Table 1. Results from our code, *e.g.* the core k_{eff} and assembly relative power distribution, are then compared against coarse mesh FDM[‡] results and coarse mesh analytic nodal method[§] results. All three results were obtained using the same X-Y-Z mesh size of 10cm x 10cm x 20cm. The core k_{eff} for this mesh configuration are 1.029096, 1.02913, and 1.029118 for PARCS, VENTURE, and our code, respectively. The assembly power distribution comparison is shown in Figure 2. Results from the coarse mesh FDM codes (VENTURE and our code) agree well. Results from the two FDM codes using the aforementioned mesh size do not agree well with results from the nodal method. To improve the accuracy of the FDM and to make the results comparable to the nodal method's results, a much smaller mesh size (~100 times smaller than the nodal method's mesh) must be employed. The mesh refinement will result in longer CPU time. Therefore, it is planned to implement a nodal method in our code in a near future.

Inner Iteration Acceleration Results: Trial-and-Error Approach

The optimum two-group ω values are determined *via* parametric study (trial-and-error approach). The number of inner iterations is based on convergence criterion of 1e-3 of RMS difference in flux value change between iteration. Identical criterion is used for each outer; hence, the number of inner iterations varies for each outer iteration. The behavior of ω values versus number of outer iterations is shown in Figure 3 while similar plot versus the CPU time is shown in Figure 4. In those two figures, the fission source extrapolation is set to 1.1 and Δk^{shift} is held at 0.04. The

[†] Benchmark problem, originally defined by B. Michaelsen (RIS0) by letter of Dec.14, 1971 to participants of the Panel on Reactor Burn-up Physics which was organized by the IAEA and held in Vienna, 12-16 July 1971.

[‡] ANL-ID.11-A1, submitted by D.R. Vondy and T.B. Fowler (ORNL) June 1, 1976. Results were obtained using VENTURE (ORNL-5062) code.

[§] H.G. Joo, et.al., PARCS (Purdue Advanced Reactor Core Simulator), A Multi-Dimensional Two Group Reactor Kinetics Code Based on the Nonlinear Analytic Nodal Method, Technical Report, PU/NE-98-26, 1998.

optimum ω values from the trial-and-error approach are 1.7 and 1.1 for the fast and thermal group, respectively. The CPU time using the best ω set is 1.01sec.

Outer Iteration Acceleration Results: Trial-and-Error Approach

The outer iteration acceleration parameters, Δk^{shift} and fission source extrapolation, are determined *via* trial-and-error approach. The fast and thermal ω values are set to 1.0. The ΔN_l is determined by the inner iteration convergence criterion of $1e-3$. The number of outer iterations as a function of Δk^{shift} and extrapolation value is shown in Figure 5. Similar plot versus CPU time is shown in Figure 6. The best CPU time from this trial-and-error approach is 1.72 seconds with $\Delta k^{\text{shift}} = 0.04$ and extrapolation value of 1.1.

Inner and Outer Accelerations Results: Automated Algorithm for Finding Optimum Values

The optimum two-group ω values and the number of inner iteration, ΔN_l , are determined *via* automated algorithm mentioned in Section 0. The optimum outer iteration acceleration parameter, *i.e.* Δk^{shift} and fission source extrapolation are determined using parametric study (Section 0) in combination with the optimum ω and ΔN_l . The optimum parameters obtained using this algorithm are as follows:

Optimum Δk^{shift}	= 0.02
Optimum fission source extrapolation	= 1.6
Optimum \mathbf{w}_{fast} and $\mathbf{w}_{\text{thermal}}$	= 1.145 and 1.068
Optimum ΔN_l fast and thermal	= 7 and 5
Optimum CPU time	= 0.814 sec.

CONCLUSIONS AND RECOMMENDATIONS

A 3-D, multi-group diffusion equation solver written in FORTRAN language and based on the finite difference method has been developed as a preliminary step in developing a full scale PWR core simulator code which can be used to perform fuel loading pattern optimization. An outer-inner iteration scheme of solving the FDM

representation of the 3-D, multi-group diffusion equation is employed. An outer iteration acceleration method using the eigenvalue shift and fission source extrapolation is implemented. The inner iteration employs the Red-Black Line SOR method. Algorithms to automatically determine the optimum ω and number of inner iteration values have also been implemented. These automated procedures relieve the user of the trial-and-error process in determining the optimum parameters for each reactor problem.

The accuracy of the coarse mesh FDM based code is known to be insufficient. Fine mesh FDM is needed to achieve acceptable fidelity; however, the computational time associated with the fine mesh FDM is tremendously larger than the time needed by a nodal method based solver for the same level of accuracy. Therefore, as a next step, a nodal expansion method will be implemented to accelerate the convergence. A Krylov based inner iteration solver will be added to replace the Red-Black Line SOR. To complete the core simulator part of the code, thermal-hydraulic feedbacks will also be added later.

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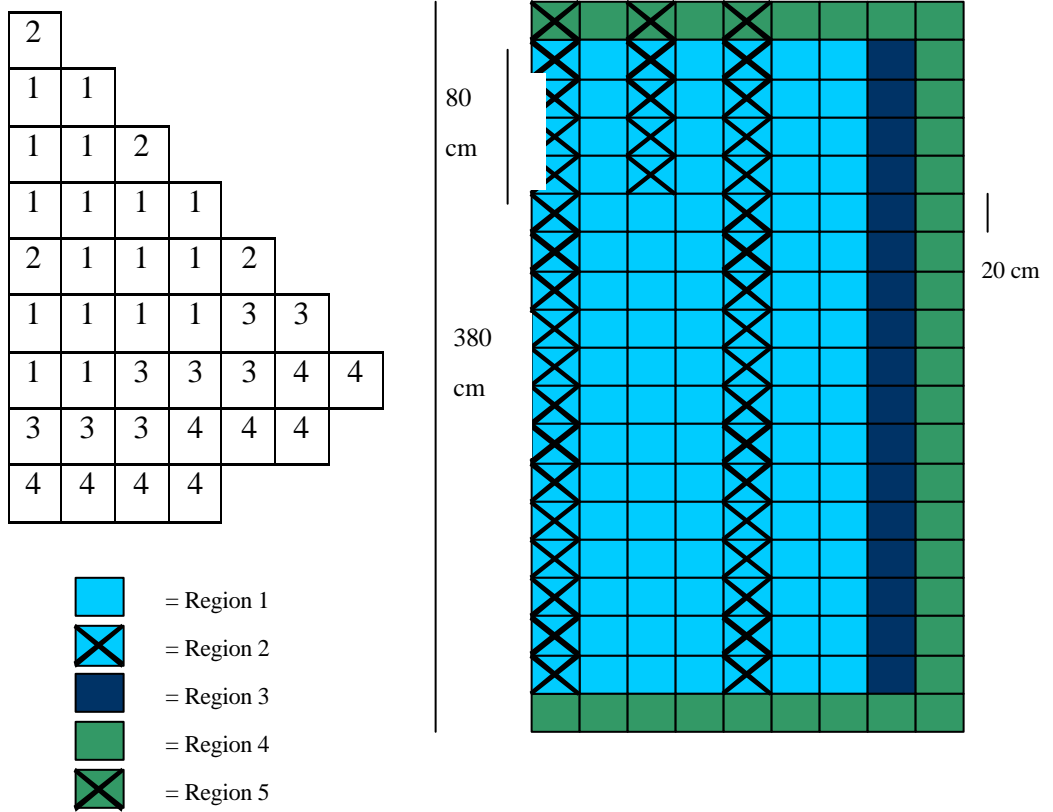


Figure 1. Three dimensional configuration for IAEA benchmark problem

Table 1. Two group constants for IAEA benchmark problem

Reg.#		D_1	D_2	S_{a1}	S_{a2}	nS_{f2}	S_{s12}
1	Fuel 1	1.5	0.4	0.01	0.085	0.135	0.02
2	Fuel 1 + Rod	1.5	0.4	0.01	0.130	0.135	0.02
3	Fuel 2	1.5	0.4	0.01	0.080	0.135	0.02
4	Reflector	2.0	0.3	0.00	0.010	0.000	0.04
5	Reflector + Rod	2.0	0.3	0.00	0.055	0.000	0.04

	1	2	3	4	5	6	7	8
A	0.7264							
	0.7356							
	0.7352							
B	1.2742	1.3899						
	1.4485	1.5604						
	1.4479	1.5596						
C	1.4156	1.4248	1.3627					
	1.5651	1.5742	1.4797					
	1.5645	1.5734	1.4784					
D	1.1884	1.2856	1.3065	1.1751				
	1.3178	1.4045	1.4035	1.2480				
	1.3174	1.4041	1.4026	1.2479				
E	0.6097	1.0685	1.1785	0.9702	0.4766			
	0.5802	1.1406	1.2226	1.0083	0.4244			
	0.5802	1.1406	1.2226	1.0084	0.4245			
F	0.9524	1.0543	1.0888	0.9238	0.7015	0.6017		
	0.9707	1.0619	1.0755	0.8970	0.6720	0.4717		
	0.9710	1.0622	1.0758	0.8973	0.6722	0.4719		
G	0.9607	0.9768	1.0016	0.8698	0.6150			
	0.9101	0.9537	0.9420	0.7383	0.4725			
	0.9105	0.9215	0.9424	0.7386	0.4727			
H	0.7798	0.7600	0.7152					
	0.6574	0.6366	0.5459					
	0.6576	0.6368	0.5461					

PARCS
VENTURE
Our Code

Figure 2. Assembly relative power distribution comparison

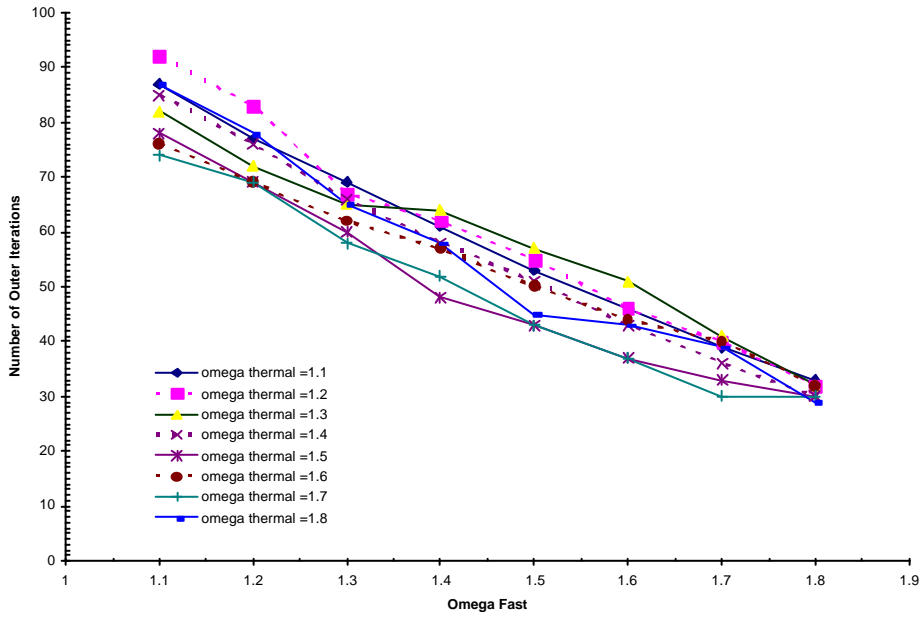


Figure 3. Omega values versus number of outer iterations

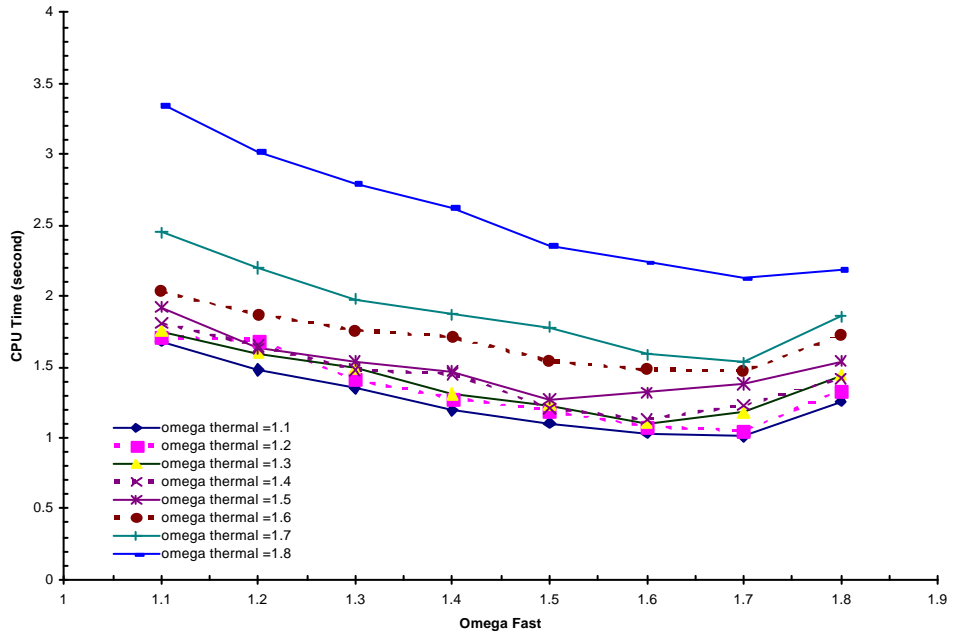


Figure 4. Omega values versus CPU time

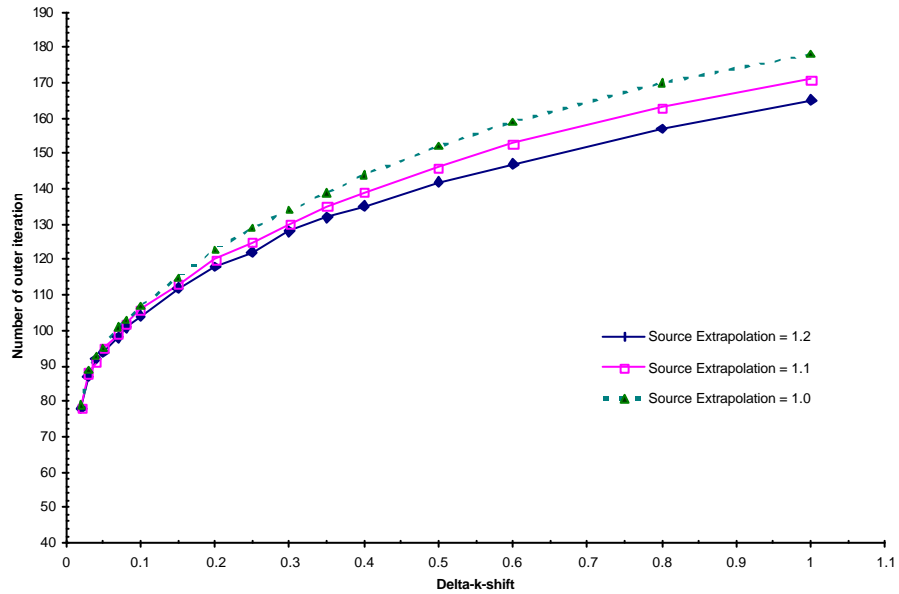


Figure 5. Eigenvalue shift and fission source extrapolation versus number of outer iterations

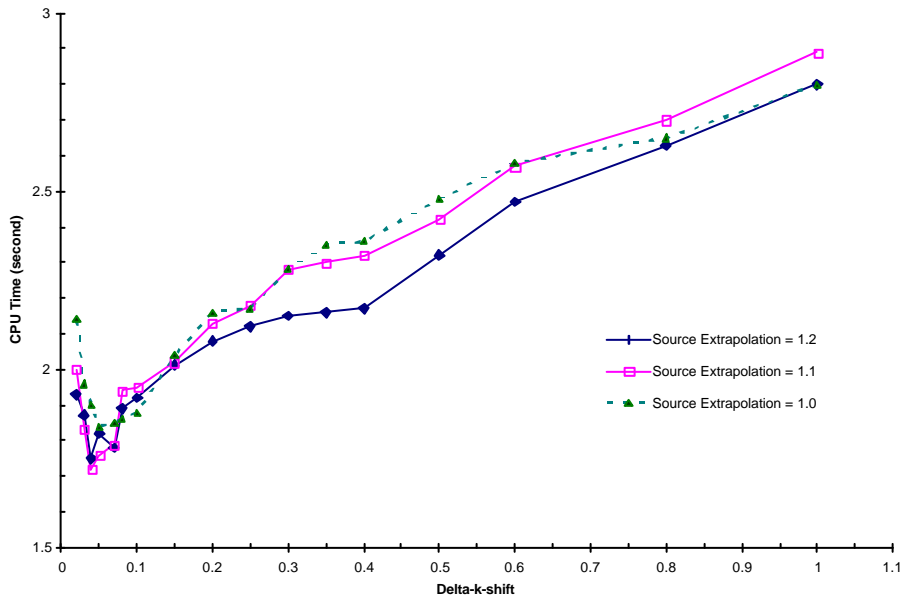


Figure 6. Eigenvalue shift and fission source extrapolation versus CPU time

DISKUSI

SYAMSA ARDISASMITA

Penyelesaian masalah persamaan difusi neutron dengan metode numerik adalah menuliskan persamaan tersebut dalam bentuk persamaan differensial dengan syarat-syarat batas dan diskretisasi domain (spatial derivatives). FDM menggunakan diskretisasi seragam (meshing) sedangkan FEM menggunakan bentuk optimal (triangulation). Bagaimana bentuk diskretisasi domain dalam Nodal Expansion Method dan apa guna dari quartic polynomial expansion?

IMELDA ARIANI

Diskretisasi domain dalam nodal expansion method yang digunakan untuk menyelesaikan persamaan difusi neutron sama dengan FDM. Quartic polinomial dalam NEM digunakan untuk menebak distribusi fluks di dalam setiap node.

ALIET YAHYA

Apakah punya data terbaru untuk FEM 3D, karena terlihat data FEM 3D untuk tahun 1976?

IMELDA ARIANI

Saat ini Finite Elemen Method sudah tidak populer di kalangan *reactor physics* untuk penyelesaian diffusion equations di Light Water Reactor. Kemungkinan besar karena geometri yang harus diselesaikan cukup sederhana (segi empat) sehingga tidak menuntut penggunaan FEM yang lebih fleksible geometrinya. Nodal method saat ini lebih /paling populer di berbagai kode komputer yang menyelesaikan 3-D persamaan difusi LWR. Karena alasan tersebut, tidak ada data perbandingan FEM vs Nodal untuk tahun-tahun yang lebih baru.

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